Physics, Nonlinear Time Series Analysis, Data Assimilation and Hyperfast Modeling of Nonlinear Ocean Waves

A. R. Osborne
Dipartimento di Fisica Generale, Università di Torino
Via Pietro Giuria 1, 10125 Torino, Italy
Phone: (+39) 11-670-7451 or (+39) 11-329-5492 fax: (39) 11-658444 email: al.osborne@gmail.it

Award #: N00014-08-1-0692

LONG-TERM GOALS

This year's report is at once a summary of the past three years of work for ONR and simultaneously a three-year plan for the future.

My new book [*Nonlinear Ocean Waves and the Inverse Scattering Transform*, Osborne, 2010] discusses the physics, nonlinear time series analysis, data assimilation and hyperfast modeling of nonlinear ocean waves. Some of the material in this book consists of mathematics not always familiar to oceanographers and may require an investment of the reader's time to take full advantage of the methods introduced there. This book, in many ways, may be compared to the book *Ocean Wave Spectra* [ONR, 1962], which was published in a revolutionary time for the field of wind waves (the 1950s and 60s). New data analysis procedures were being developed by Pierson, Longuet-Higgins, Munk, Hasselmann and others. The concept of the power spectrum was quite new to physical oceanographers. It is useful in this context to recall the work of Paley, Weiner and Rice in the 1930s and 1940s and the subsequent application to power spectra and wind waves in the 1950s and 1960s: This work was based upon the integrable of the square root of dx! I recall well the consternation of physical oceanographers at that time about this seemingly impossibly difficult mathematics (see Kinsmann's book for aid in understanding what was at that time the new mathematics). Likewise the introduction of the Hasselmann equation in 1961 was based on the derivation of kinetic equations from the Euler equations, also rather esoteric mathematics at that time. Now of course these areas of mathematics have been absorbed into the mainstream of wind/wave research and have lead to the development of modern forecasting and hindcasting models. Indeed the mathematics of 1960 seems mainstream today.

The methods in [Osborne, 2010a] should be thought of in this way. Some initial investment in the mathematical methods will pay great dividends in future years for new physical understanding, new time series analysis methods, and hyperfast modeling efforts which are already yielding computer codes thousands of time faster on a single core and substantially more on multiple cores.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
2010		**00-00-2010 to 00-00-2010**

4. TITLE AND SUBTITLE **Physics, Nonlinear Time Series Analysis, Data Assimilation and Hyperfast Modeling of Nonlinear Ocean Waves**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Universit?i Torino,Dipartimento di Fisica Generale,Via Pietro Giuria 1,10125 Torino, Italy,**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**Same as Report (SAR)**	**7**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

OBJECTIVES

I now give a brief summary of present and future objectives.

Rogue Waves - I am presently writing a new book on *rogue waves* [*Rogue Waves and Holes in the Sea*, Osborne, 2011], that is a general treatise on nonlinear waves that discusses how discrete *coherent structures* appear in ocean waves above a certain threshold, creating *rogue wave packets* (see Figs. 1-3). These rogue packets are exact solutions of the nonlinear Schroedinger equation and are approximate solutions of the Euler equations. The packets have been found in a number of new laboratory and ocean experiments and new methods for both (1) time series analysis and the (2) analysis of time series arrays have been developed to determine when rogue packets occur in a sea state. This book will also give a comprehensive overview of hyperfast modeling and nonlinear Fourier analysis of the Euler equations. An additional topic of the book will be the *real time rapid environmental assessment* of rogue wave conditions: (1) ship board *nowcasting* capability for the determination of the presence of rogue waves in a present sea state and (2) wind/wave hindcasting and forecasting methods for computation by, say, Fleet Numerical. These methods are based upon extending the methods of Osborne [2010a,b] for the Euler equations.

Hyperfast Coastal Dynamics – The hyperfast method is being used to develop numerical methods for the Boussinesq equations in shallow water. Preliminary versions of the code already exist and show considerable promise for future coastal dynamics work.

Hyperfast Euler Equations – The new methods are being applied to the full Euler equations for surface and internal waves. Wind-wave coupling will also be included in this effort. Essentially the Euler equations algorithm will be a generalization of the Boussinesq algorithm, with extensions out to fifth order, similar to higher order methods (HOM) used over the past 25 years. The difference is that the codes developed herein are about 500 P times faster than the HOMs, where P is the number of available cores in a computer.

Hyperfast Navier-Stokes Equations – The hyperfast algorithm for the Navier-Stokes equations has presented a real challenge and preliminary results are already at hand. The approach is being developed with arbitrary boundaries, including continental and bottom boundary conditions. Coupling the ocean to the atmosphere is also underway. Ocean models of this type would lead the way to future modeling efforts of the ocean/atmosphere system.

Hyperfast Computational Fluid Dynamics – HyperCFD numerical methods are also being presently developed. The hyperfast method will be applied to problems at the order of the Navier-Stokes equations with special boundary conditions in order to provide for the design of offshore structures, ships, submarines, etc.

Computer programs are being developed for all of the above models and data analysis algorithms and will be written up in a third book: *Nonlinear Hyperfast Modeling of Hydrodynamical Systems* [Osborne, 2012].

APPROACH

Hyperfast models for the so-called *isospectral* or *integrable* equations are developed on a straightforward basis [Osborne, 2010]. I discuss here hyperfast modeling of the KP equation as an example of the use of the method.

We first consider the Kadomtsev-Petviashvili (KP) equation

$$\eta_t + c_o\eta_x + \alpha\eta\eta_x + \beta\eta_{xxx} + \gamma\ \partial_x^{-1}\eta_{yy} = 0 \tag{1}$$

Here $c_o, \alpha, \beta, \gamma$ are constant coefficients and $\eta(x, y, t)$ is the wave amplitude as a function of the two spatial variables, x, y and time, *t*. The KP equation (1) is a natural two-space-dimensional extension of the KdV equation. The periodic KP solutions include *directional spreading* in the wave field:

$$\eta(x,t) = 2\frac{\partial^2}{\partial x^2}\ln\theta(x, y, t\,|\mathbf{B}, \phi) \tag{2}$$

Here the generalized Fourier series has the form given in (4) below, where the phase has the *two dimensional* expression:

$$\mathbf{X}(x, y, t) = \mathbf{k}x + \mathbf{l}y - \omega t + \phi \tag{3}$$

The spatial terms include both the *x* and *y* coordinates, $\mathbf{k}x$ and $\mathbf{l}y$, which allows wave spreading to be taken into account. The KP equation is the first nonlinear step toward a directional sea state; KP is however limited to small directional spreading. Improving the directional spreading characteristics of the KP equation requires the addition of physically important corrections [Osborne, 2010].

The *generalized Fourier series*, $\theta(x, t\,|\mathbf{B}, \phi)$, is given by the expression

$$\theta(x, y, t\,|\,\mathbf{B}, \phi) = \sum_{m_1=-\infty}^{\infty}\sum_{m_2=-\infty}^{\infty}\cdots\sum_{m_N=-\infty}^{\infty} e^{i\sum_{n=1}^{N} m_n X_n - \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} m_m m_n B_{mn}} \tag{4}$$

where $X_n = k_n x + l_n y - \omega_n t + \phi_n$. The function $\theta(x, y, t\,|\mathbf{B}, \phi)$ is also called a *Riemann theta function* or *multidimensional Fourier series*. Here **B** is the Riemann matrix (the "spectrum" of the solution), the vectors **k**, **l** constitute the usual wavenumbers, the vector ω contains the frequencies and the vector ϕ forms the phases. The *inverse problem* associated with (2), (3) allows one to determine the Riemann matrix, wavenumbers, frequencies and phases appropriate for solving the *Cauchy problem* for KP: Given the spatial variation of the solutions at $t = 0$, $\eta(x, y, 0)$, compute the solution for all time, $\eta(x, y, t)$. This is a necessary step for the numerical simulations presented herein. The *solitons, Stokes waves and sine waves lie on the diagonal of the Riemann matrix*; the *off-diagonal terms contain the nonlinear interactions*.

Why is the above approach useful for hyperfast numerical simulations? Because the Riemann theta function can be programmed as a *fast theta function transform* (FTFT), just as the Fourier transform can be programmed as a *fast Fourier transform* (FFT). Therefore the numerical integration of KP (1) can be evaluated at specific time points, necessary only for graphical purposes or for extracting useful (often statistical) properties of the sea surface. This contrasts to the FFT that must be evaluated at very small time steps when used for the numerical integration of a nonlinear partial differential equation. This is one reason why the *higher order methods* require considerable amounts of computer time.

WORK COMPLETED

Recent work has emphasized the forecasting and hindcasting of rogue waves. The algorithms are based to leading order on the nonlinear Schroedinger type equations and to high order based on the Euler equations. Essentially the ideas proceed in the following way. The largest rogue waves have been demonstrated to arise because of the presence of nonlinearities in the Euler equations. Fundamental knowledge of the instabilities (such as that due to the Benjamin-Feir instability) of the equations of motion has been found to give rise to “unstable mode” rogue waves. Indeed the instability diagram in wavenumber space has been computed using the modern methods and have been related to the generalization of inverse scattering theory. Thus in all of this work nonlinear Fourier methods based upon the inverse scattering transform are fundamental. They tell us about the nonlinear Fourier structure of the waves. An example of the hindcasting method is given in the next section.

RESULTS

I now discuss the hindcasting result given in Figures 1-3. Winds from an energetic storm have been used to generate the directional spectrum shown in Fig. 1. The directional spectrum is a function of frequency and direction as indicated in the figure. Here the significant wave height is over 8 m and the peak period is 12.5 sec. Using the inverse scattering transform I have found that unstable rogue modes occur above a particular threshold as shown in the figure. This effect physically happens because of the formation of pairs of Stokes waves that phase lock with each other above the threshold. The paired Stokes waves form packets that are unstable to perturbations of their envelopes. Figure 2 shows the power spectrum, found by integrating over the angle in the directional spectrum. The rogue packets are indicated as red dots in the figure. Here I have graphed the maximum amplitudes of the central wave in the packet. As can be seen the maximum packet amplitudes far exceed (by about a factor of three) the amplitudes of the background sine or Stokes waves in the spectrum. It is therefore easy to see how the rogue packets can rise up out of the background sea state so easily. In Fig. 3 I show the largest of the two-dimensional rogue packets in the spectrum. One sees not only that the crest height is quite large in the present case. In addition the trough forms a very interesting hole in the se. The details of the hindcasting and forecasting work will be given in future papers and in Osborne [2011].

TRANSITIONS

Transitions expected are related to the use of the codes as guidance to ships and unmanned, unteathered vehicles in the internal wave field and for the real time sampling of the environment, including the acoustic waves.

RELATED PROJECTS

An intimate relationship between our results and other projects exists because the sea surface provides a major forcing input to many kinds of offshore activities, including the dynamics of floating and drilling vessels, barges, risers and tethered vehicles. The present work leads to a nonlinear representation of the sea surface forcing and vessel response for shallow water waves.

REFERENCES

Osborne, A. R., *Nonlinear Ocean Waves and the Inverse Scattering Transform* (Academic Press, 2010, 976 pages).

Osborne, A. R., Rogue Waves: Classification, measurement and data analysis, and hyperfast modeling, *Eur. Phys. J. Special Topics*, **185**, pp. 225-245, 2010.

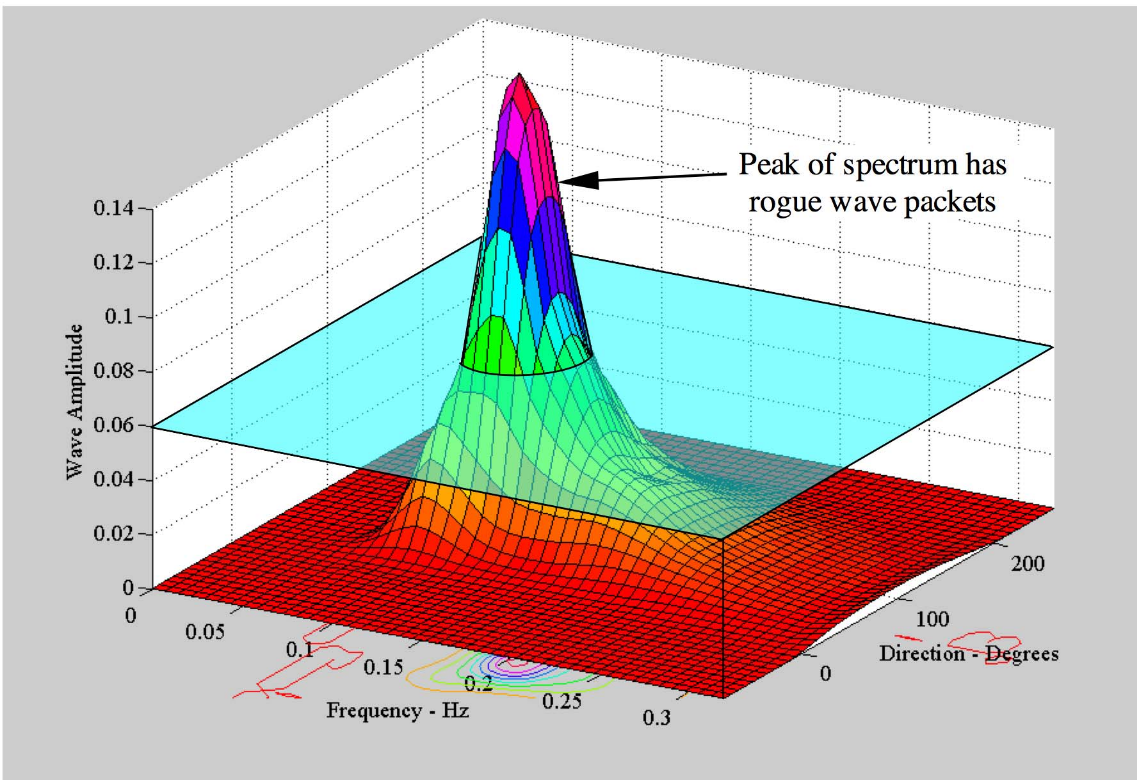

Figure 1. The directional wave spectrum from a hindcast storm. The significant wave height was over 8 meters. Nonlinear Fourier analysis demonstrates the existence of a threshold (the horizontal cyan plane) above which rogue wave packets form. This work illustrates the power of the nonlinear Fourier method for forecasting/hindcasting rogue waves.

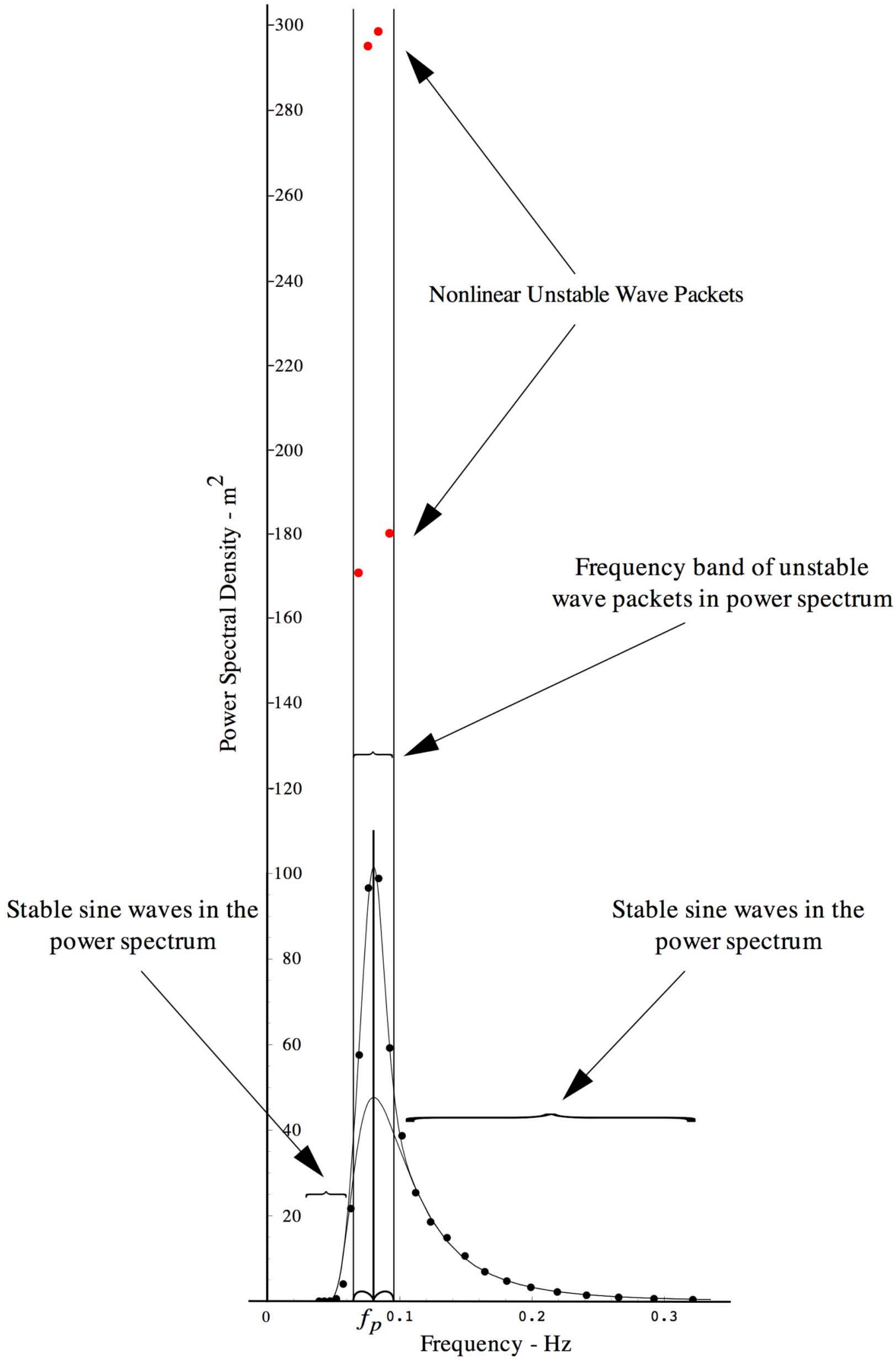

Figure 2. The power spectrum, in the hindcast case of Figure 1, forms rogue wave packets about the peak of the spectrum. The rogue packets are indicated by red dots that have been graphed with their maximum physical amplitude. Spectral components outside the central rogue wave band are Stokes or sine waves.

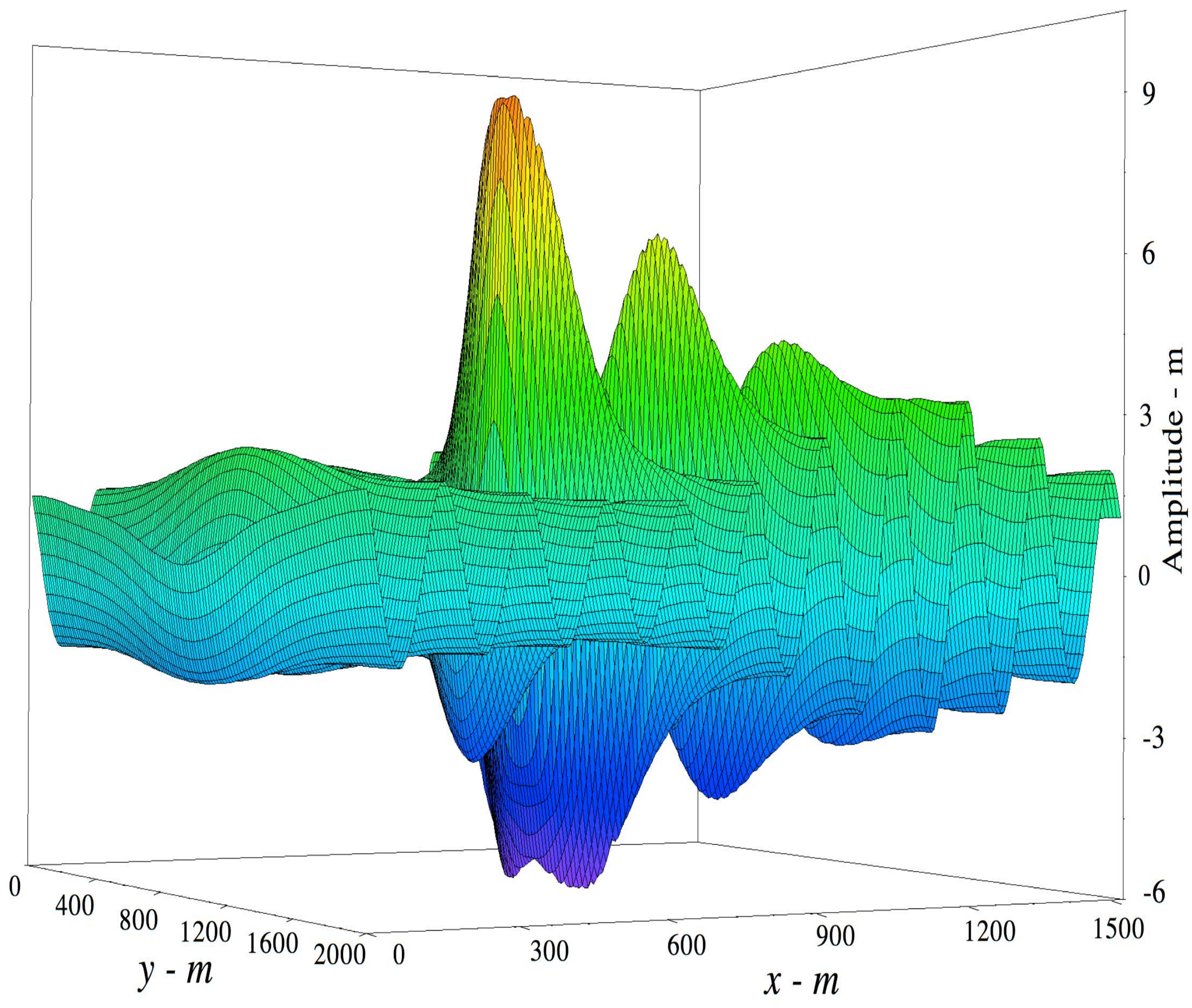

Figure 3. Largest rogue packet from the peak of the hindcast spectrum of Figure 1.